Summer Calculus Assignment Featuring: The Required Algebra / Geometry / Pre-Calculus skills you should have learned.

Directions: ***ALL*** *problems must be solved algebraically and graphed* ***<u>without</u>*** *the aid of a graphing calculator. A non-graphing calculator may be used on any problem.*
ALL *answers must be reproducible from the work shown.*

Part I

$f(x) = 8x^2 - 180x - 1100$ $\quad$ $g(x) = -13x^2 - 75x + 2050$

1) Find where f(x) =500

2) Find the vertical distance between f(x) and g(x) when x = 5.

3) Find where f(x) = g(x). What verbal description can you give to the solution(s)?

4) Find where f(x)>0, f(x)<0, and f(x)=0.

5) Find all the points where f(x) has a maximum or a minimum.

6) Find all intervals where f(x) is increasing and decreasing.

7) Find where g(x) is positive, g(x) is negative, and the roots of g(x).

8) Find all the points where g(x) has a maximum or a minimum.

9) Find all intervals where g(x) is increasing and decreasing.

10) Using g(x) make a table of values where x is the even consecutive integers from x = -4 to x = 8. Graph this table and connect each consecutive value with a line segment. Draw vertical line segments from each point to the x-axis to make trapezoids. Calculate the sum of all the areas of these trapezoids. Calculate the slope of each non-horizontal leg of every trapezoid.

11) Read this entire paragraph before starting this problem. Graph both f(x) and g(x) on the same axis. Shade the region in the first quadrant bound by these functions. Draw a vertical line through the region at x = 5. Imagine a square whose edge is on x=c that is bound by both the functions but the square stands off of the x-y plane in the third dimension like a brick wall in the middle of a field.

a) Draw the square and the plane in a 3-D perspective.

b) What is the perimeter of the square?

c) What is the area?

Part II

R(x) = $\frac{x^3+2x^2-33x-90}{x^2+3x-10}$

12) What is the domain and range of the rational function R(x)?

13) What happens to the function as x approaches infinity? …as x approaches negative infinity?

14) Find the location of any holes, a.k.a. removable discontinuities.

15) Give the equations for any vertical or horizontal asymptotes.

16) Describe what happens to the values of R(x) as x approaches each vertical asymptote from the left.

17) Describe what happens to the values of R(x) as x approaches each vertical asymptote from the right.

18) Using the information from Problems 12 – 17, Draw an approximate graph of R(x). DO NOT USE A GRAPHING CLACULATOR! Your graph should represent your answers from Problems 12 – 17.

Part III

$$h(x) = \begin{cases} x^2 + 2x & x < 0 \\ \sin(x) & 0 \le x < \frac{3\pi}{2} \\ \tan\left(x - \frac{3\pi}{2}\right) - 1 & x \ge \frac{3\pi}{2} \end{cases}$$

19) Graph the piece-wise function h(x) from x = -5 to x = 3π. (Yes, without a graphing calculator!)

20) Is the function continuous or discontinuous? Explain how you know.

Part IV

21) Graph the following circle. $(x-2)^2 + (y+4)^2 = 25$

22) Find all points where $x = -1$.

23) Give the equations of the tangent lines to the circle at these points. (Hint: a tangent is perpendicular to a radius at the point of tangency.)

Part V

23) Draw the graph of the line 3x+4y=10.

24) Determine the slope, the x – intercept, and the y – intercept.

25) Revolve the line segment that is completely bound in the first quadrant around the y-axis to form a cone. Draw this cone and calculate the volume and total surface area.

26) On another graph rotate the same line segment around the x-axis to form a different cone. Draw this cone and calculate the volume and total surface area.

Part VI (Challenging Problems)

27) Find all values of x satisfying the equation $\frac{x-\sqrt{x+1}}{x+\sqrt{x+1}} = \frac{11}{5}$.

28) Find the real values of x such that $3^{2x^2-7x+3} = 4^{x^2-x-6}$.

29) Find the real values of x such that $xlog_2 3 = log_{10} 3$.

30) Two trains, each traveling uniformly at 50 m.p.h, start toward each other, at the same time, from stations A and B, 10 miles apart. Simultaneously, a super powered bee starts from station A, flying parallel to the track at a uniform speed of 70 m.p.h., toward the train from station B. Upon reaching the train it comes to rest, and allows itself to be transported back to the point where the trains pass each other. Find the total distance the bee travels.

31) Two trains, one 350 feet long, the other 450 feet long, on parallel tracks, can pass each other completely in 8 seconds when moving in opposite directions. When moving in the same direction, the faster train completely passes the slower one in 16 seconds. Find the speed of the slower train.